IMPACT OF SUCTION AND INJECTION ON HYDROMAGNETIC OSCILLATORY PLAIN POISEUILLE FLOW IN POROUS MEDIUM

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Abstract

This paper provides a theoretical investigation of a Plain Poiseuille flow in presence of magnetic field through porous medium confined between two stationary parallel horizontal porous plates, where the flow is driven by an external pressure gradient. The velocity at the boundary is zero and increases to a maximum at the axis of the horizontal channel. The flow is resisted by shear stresses in the fluid. Suction and injection have been applied naturally. The governing equations and the relevant boundary conditions are converted into non-dimensional form using non- dimensional quantities and thus being solved by using closed form analytical technique and the results are discussed with the help of appropriate graphs. The outcomes of disparity of various parameters on the velocity field and shear stress are discussed. The result shows that, when there is a rise in Darcy number (D_a) or suction/injection parameter (S), there is a drop in the fluid velocity profiles. A rise in D_a or S is found to escalate the shear stress(τ). Oscillatory flow finds a wide range of applications in chemical, biochemical and process engineering.

Key words: Plain Poiseuille flow, Oscillatory flow, Porous medium, Magnetic drag force, Injection and suction, Shear stress.

1. INTRODUCTION

Investigation of MHD flows are significant due to its vast applications in different areas of research like MHD generators, MHD pumps, MHD flowmeters, etc. An all-inclusive literature on different characteristics of physical situations based on the MHD channel flow problems can be found in [3] and [9]. Eckert [5] acquired the precise solution of Navier-Stokes equations for the fluid flow confined between two parallel porous plates subject to constant injection/suction.

Dash and Ojha [4] investigate theoretically flow of a viscoelastic fluid of second order with heat transfer in presence of MHD confined between two infinite parallel plates through a porous medium. Mehmood et al. [8] investigated the slip effects of MHD oscillatory fluid flow through a porous medium in presence of heat transfer. Cookey et al. [2] studied the problem of free convection and oscillatory Couette flow in presence of transverse magnetic field in a porous medium bounded by two horizontal parallel porous walls where the lower plate being in motion while upper plate being stationary. Seth et al. [10] studied unsteady Couette flow problem in presence of transverse magnetic field when the fluid flow is restricted to porous boundaries. Baag et al. [1] studied MHD free convection flow of Walters

(Model B) - viscoelastic fluid with time dependent suction through porous medium in presence of radiative heat transfer. Falade et al. [6] investigated the effect of unsteady oscillatory flow subject to suction/injection in presence of transverse magnetic field through a permeable vertical channel filled with a porous medium with non-uniform wall temperature. More recently, Hezekiah et al. [7] investigated theoretically the effects of different parameters on convective flow in presence of MHD through a porous medium with thermal radiation and chemical reaction. Channel flow in a chemically-reacting fluid between two long vertical parallel flat plates in the presence of a transverse magnetic field is presented by Ahmed et al. [11]. Recently, the analysis of heat and mass transfer of nanofluids in a channel and a stretching sheet for the application of magnetic drag forces have been presented by Hazarika et al. [12 – 15].

In this paper, the aim is to analyze the impact of suction and injection on the oscillatory Plain Poiseuille flow in presence of magnetic drag force confined between two stationary infinite parallel porous plates through a porous medium. It is also included the significant effect of Darcy number over the fluid velocity and shear stresses.

2. MATHEMATICAL FORMULATION

We consider a two-dimensional Plain Poiseuille flow of the assumed fluid through a saturated porous medium confined between two horizontal porous plates of semi-infinite length. The plates are situated at a distant 'd' apart. Here we have selected a rectangular coordinate system such that x' lies along the center of the channel and y' is taken normal to x'. The flow is assumed to be in x' direction. A uniform magnetic field H_o is applied normal to the plate. The constant injection and suction velocity has been applied normally to the lower plate and the upper plate respectively as shown in **Figure 1**.

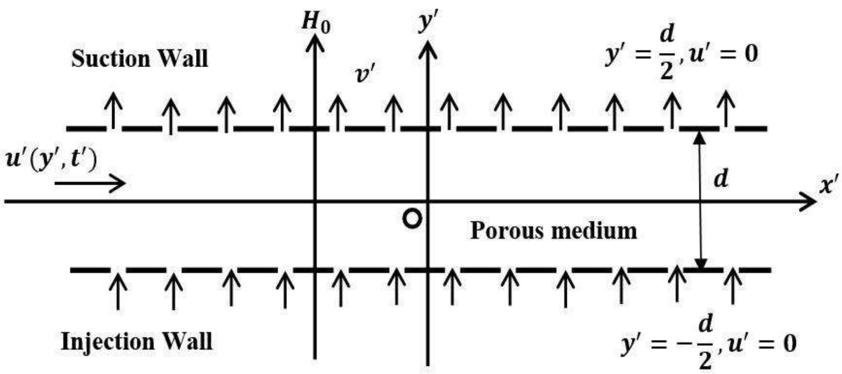

Figure 1: Flow Geometry of the problem

Due to infinite length of surface, all the physical properties of the fluid are independent of x' except the pressure gradient. So, due to the time dependence of the pressure, the flow becomes oscillatory. Plane Poiseuille flow is flow created between two infinitely long parallel plates, separated by a distance d with a constant pressure gradient is applied in the direction of flow. The flow is essentially unidirectional because of infinite length. The Conservation mass and Navier–Stokes equations takes the form:

Equation of Mass:

$$\frac{\partial v'}{\partial y'} = 0 \tag{2.1}$$

Equation of Momentum (Navier-Stokes Equation for Velocity):

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma H_0^2}{\rho}u' - \frac{\nu}{K'} \tag{2.2}$$

The relevant boundary conditions are

$$\left\{\begin{matrix} u' = 0\,, \quad v' = \mathrm{V}\,, \ \text{at} \ y' = \frac{d}{2} \\ u' = 0\,, \ v' = \mathrm{V}\,, \ \text{at} \ y' = -\frac{d}{2} \end{matrix}\right\} \tag{2.3}$$

With the introduction of the following non-dimensional parameters,

$$\left\{\begin{matrix} x = \frac{x'}{d}\,, \ y = \frac{y'}{d}\,, \ u = \frac{u'}{d}\,, \ t = \frac{t'\nu}{d}\,, \ p = \frac{p'}{\rho \mathrm{V}^2}\,, \\ S = \frac{Vd}{\nu}\,, \ D_a = \frac{K'V}{\nu d}\,, \ H_a = dH_0\sqrt{\frac{\sigma}{\mu}} \end{matrix}\right\} \tag{2.4}$$

On using (2.4), the equation (2.2) becomes,

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{S}\frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{S}{H_a}^2 + \frac{1}{D_a}\right)u \tag{2.5}$$

Notations: ρ = density, t' = time, u' = axial velocity, v'= transverse velocity, ν = kinematic viscosity, p' = pressure gradient, K'= constant of permeability of the porous medium, H_0 = uniform magnetic field, μ = dynamic viscosity, σ = electrical conductivity, S = injection/suction parameter, D_a = Darcy number and H_a = Hartmann number/magnetic drag force.

The boundary conditions are transformed to

$$\left\{\begin{matrix} u = 0\,, \ v = \mathrm{V}\,, \ \text{at} \ y = \frac{1}{2} \\ u = 0\,, \ v = \mathrm{V}\,, \ \text{at} \ y = -\frac{1}{2} \end{matrix}\right\} \tag{2.6}$$

3. METHOD OF SOLUTION

To solve the equation (2.5) with the help of equation (2.6), we structure the solution as below:

$$u(y,t) = \bar{u}(y)e^{i\omega t}\ , \quad -\frac{\partial p}{\partial x} = \lambda e^{it} \tag{3.1}$$

where λ = constant and ω = frequency of oscillations.

Putting (3.1) into equations (2.5), we get

$$\bar{u}'' - S\bar{u}' - n^2\bar{u} = -S\lambda \tag{3.2}$$

where $n = \sqrt{H_a^2 + \frac{S}{D_a} + i\omega S}$.

The relevant transformed boundary conditions are

$$\bar{u} = 0\ ,\ \text{at}\ y = \frac{1}{2} \quad \text{and} \quad \bar{u} = 0\ ,\ \text{at}\ y = -\frac{1}{2} \tag{3.3}$$

On using (3.3), the equation (3.2) gives the fluid velocity, $u(y,t)$, as:

$$u(y,t) = \frac{S\lambda}{n^2}\left[1 - \frac{cosh(Ay)}{cosh(\frac{A}{2})}\right]e^{i\omega t} \ \text{where}\ A = \frac{1}{2}\left[\frac{S + \sqrt{S^2 + 4n^2}}{2}\right] \tag{3.4}$$

The shear stress (τ) at the lower plate $y = -1/2$ is given by:

$$\tau = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=-1/2} = \frac{\lambda S}{n^2}tanh\left(\frac{A}{2}\right) \tag{3.5}$$

4. RESULTS AND DISCUSSION

In this investigation, the significant results of disparity of the aforesaid different parameters are presented graphically in Figures 2 to 5.

In **Figure 2,** velocity distribution (u) for Hartmann number (H_a) and Darcy number (D_a) for different values of $\omega = 10, S = 0.2, \lambda = 10$ is shown. It is seen that u is a decreasing function of Hartmann number (H_a) and increasing function of Darcy number (D_a). Velocity peak is observed in the middle of the channel for all profiles. No back flow is experienced throughout the channel. The flow is fully developed and therefore, there is axisymmetric flow about the x-axis (centre line of symmetry).

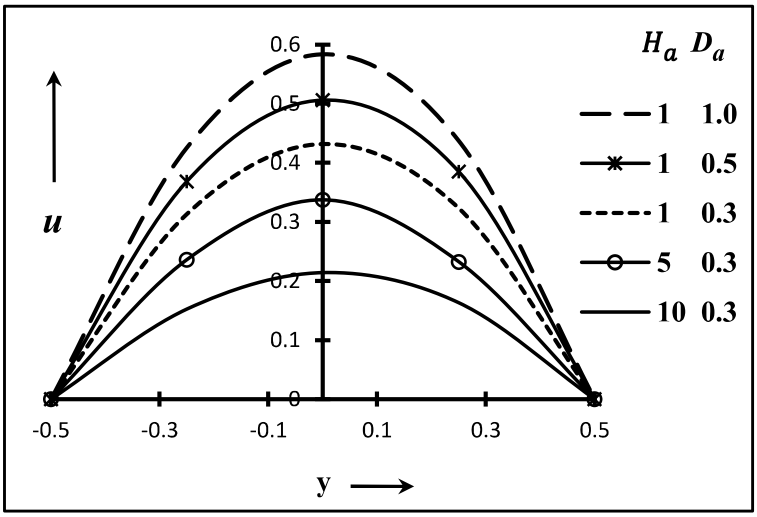

Fig 2: Velocity distributions for H_a and D_a at $t = 0$.

In **Figure 3,** velocity distribution (u) for injection/suction parameter (S) and frequency of oscillations (ω) for different values of $H_a = 5\,, D_a = 0.5\,, \lambda = 10$ is presented. The frequency of oscillations (ω) declined the flow velocity (u) and it is escalated by the application of injection/suction parameter (S). Maximum velocity is noticed in the middle of the channel for higher values of S and minimum has marked at the lower ω. Due to physical point of view in Poiseuille flow, maximum velocity of fluid particles is attained at the axis of the channel and velocity diminishes towards the surface of the channel.

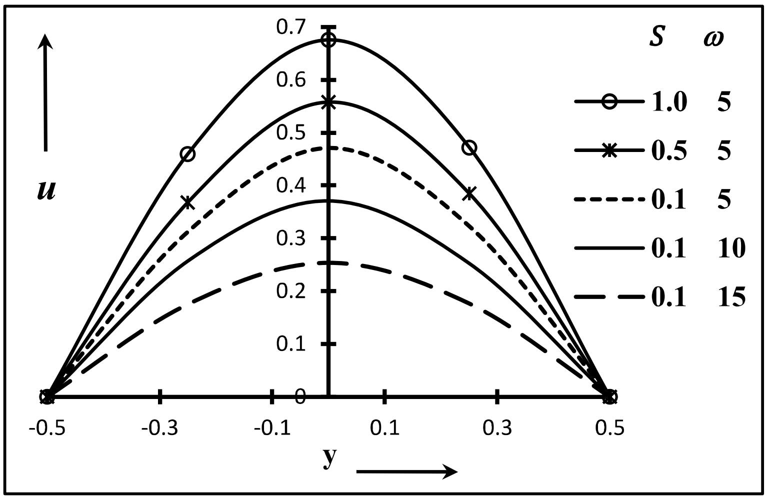

Fig 3: Velocity distributions for S and ω at $t = 0$.

In **Figure 4** variations of skin friction (τ) for Magnetic field (H_a) and injection/suction parameter (S) at the lower plate $y = -1/2$ for different values of $D_a = 0.5\,, \lambda = 10$ is illustrated. Clearly all the profiles of τ decay as H_a increases since larger Hartmann number corresponds to greater magnetic drag force, which resists the movement of the fluid particles in the boundary layer. Flow reversal is observed i.e. shear stress becomes negative for large $H_a = 5{,}10$ at ω (> 4.8). Inspection shows that increasing S accelerates the flow i.e. enhanced the shear stresses, to the extent that for $S = 1.0$. An increase in ω also strongly reduces the skin friction, in consistency with earlier discussion for the velocity response (Figure 3).

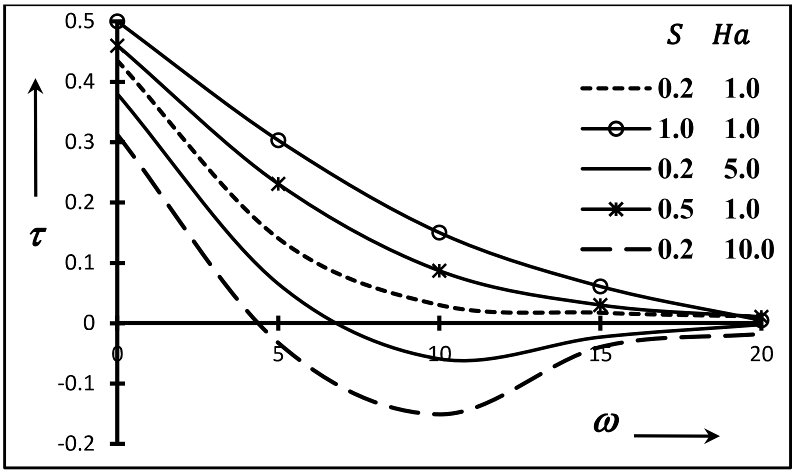

Fig 4: Variations of τ for H_a and S versus ω at $t = 0.2$.

In **Figure 5**, distribution of shear stress at the lower plate $y = -1/2$ for Darcy number (D_a) and frequency of oscillations (ω) for different values of $H_a = 3$, $S = 0.5$, $\lambda = 10$ is depicted. The shear stress is reduced at the lower plate throughout the channel that means shear stresses are the decreasing function of frequency. For all $\omega > 4.8$, back flow is sustained throughout the regime (for small D_a). Rising D_a enhanced shear stresses and large Darcy number gives maximum numerical values of τ.

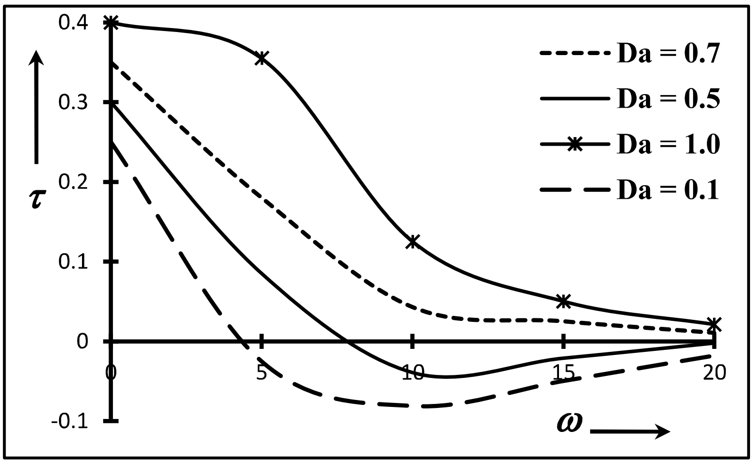

Fig 5: Variations of τ for D_a versus ω at $t = 0.2$.

5. CONCLUSIONS

Through the outcomes of this investigation, the significant results are:

- The growth of suction/injection parameter enhances the flow velocity and its related boundary layer significantly.
- The fluid velocity profiles are decelerated with increase in H_a.
- H_a declines the shearing stresses.
- D_a enhances the fluid velocity.
- Velocity overshoot has observed in the middle of the horizontal channel for the response of D_a and S.
- Shear rates is controlled entirely by the oscillations of fluid particles.

- The flow is fully developed and laminar and hence the velocity profiles are Parabolic.
- Within the inlet length, the velocity profile changes in the direction of the flow and the fluid accelerates or decelerates as it flows.
- The flow is balance by the pressure, viscous, and inertia forces of the fluid molecules.
- The Poiseuille flow is very important in hemorheology and hemodynamics, both fields of physiology.

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